

Renormalizable Sectors in Resonance Chiral Theory: $S \rightarrow \pi\pi$ Decay Amplitude

L.Y. Xiao and J.J. Sanz-Cillero

*Department of Physics, Peking University, Beijing 100871, P.R. China**

(Dated: February 1, 2008)

We develop a resonance chiral theory without any *a priori* limitation on the number of derivatives in the hadronic operators. Through an exhaustive analysis of the resonance lagrangian and by means of field redefinitions, we find that the number of independent operator contributing to the $S \rightarrow \pi\pi$ decay amplitude is finite: there is only one single-trace operator (the c_d term) and three multi-trace terms. The deep implication of this fact is that the ultraviolet divergences that appear in this amplitude at the loop level can only appear through these chiral invariant structures. Hence, a renormalization of these couplings renders the amplitude finite.

PACS numbers: 11.15.Pg, 12.39.Fe

Introduction

One of the most important issues in the construction of hadronic lagrangians is how to restrict the number of operators that contribute to a given matrix element. The underlying symmetries of Quantum Chromodynamics (QCD) constrain the structure of the action [1] but the number of allowed symmetry invariant terms is, in general, infinite.

At low energies, it is possible to describe the interaction of the Goldstones from the spontaneous chiral symmetry breaking through an effective field theory based on chiral symmetry, namely, Chiral Perturbation Theory (χ PT) [2, 3, 4]. Although one may construct an infinite number of operators, it is possible to organise the lagrangian in terms of increasing number of derivatives, where the dominant contribution to the low-energy amplitudes is provided by the lowest-order operators.

As the energy is increased, heavier degrees of freedom (the mesonic resonances) need to be included and the expansion in powers of momenta breaks down. All the resonance operators are equally important, independently of their number of derivatives. Heuristically, it is possible to argue that terms with higher derivatives spoil the asymptotic short-distance behaviour of the QCD matrix elements [5, 6]. Although it has provided successful phenomenological determinations [5, 6, 7, 8, 9], this “lowest-derivative-number” rule still lacks of solid theoretical foundations and, eventually, terms with higher derivatives may also need to be considered [10]. Nonetheless, the crucial point to develop a resonance field theory is to prove that for the description of any amplitude there is always a finite number of independent operators that enter into play. A deep implication that stems from the existence of such minimal basis of operators is that the structure of chiral-invariant ultraviolet divergences arising at the loop level [11] must be also contained in this basis and, therefore, the number of counter-terms for a definite amplitude is finite.

In this note, we analyse the general structure of a chiral invariant theory for resonances where no *a priori* re-

striction is made on the number of derivatives or resonance fields in the operators. The large number of colours limit [12, 13] provides a convenient perturbative expansion in powers of $1/N_C$, where loops appear at subleading orders [8, 9, 11]. We focus the attention on the terms of the lagrangian that contribute to the $S \rightarrow \pi\pi$ decay amplitude and we find the corresponding minimal basis of operators by means of field redefinitions.

We want to stress again that the departing lagrangian is completely general and no assumption is made on the number of derivatives in the terms of the lagrangian. No simplification is made on the part of the action responsible of other amplitudes, which remains general all along the letter. If a more complicate matrix element is to be computed, one should provide again theoretical arguments to forbid higher derivative operators, independently of whether the present $S \rightarrow \pi\pi$ simplifications are taken into account or not.

Building blocks of a chiral invariant action

A non-linear realization of the chiral Goldstone bosons is considered, being described in the coset space by the coordinates $\pi = \sum_a \frac{1}{\sqrt{2}} \lambda_a \pi_a$. We choose the canonical coset representatives $(\xi_L(\pi), \xi_R(\pi))$ such that $\xi_R(\pi) = \xi_L^\dagger(\pi) \equiv u(\pi)$ [4]. The latter transforms under the chiral group G in the way,

$$u(\pi) \xrightarrow{g \in G} g_R u h^\dagger = h u g_L^\dagger, \quad (1)$$

with the exponential realization $u = \exp \{i\pi/\sqrt{2} F\}$ and the chiral transformation $g = (g_L, g_R) \in G$. The compensator field $h = h(g, \pi)$ depends both on the chiral transformation and the Goldstone fields [4].

The building blocks of our resonance lagrangian will be hadronic tensors transforming covariantly under chiral transformations:

$$X \xrightarrow{g \in G} h X h^\dagger. \quad (2)$$

We choose a representation where the $q\bar{q}$ resonance multiplets S, V, \dots transform in this way [5, 10]. The Goldstone

fields will enter in the lagrangian through the basic covariant tensors [4],

$$\begin{aligned} u_\mu &= i \{ u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i \ell_\mu) u^\dagger \}, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \end{aligned} \quad (3)$$

or covariant derivatives $\nabla^\rho \nabla^\sigma \dots$ of them. The field $\chi = 2B_0(s + ip)$ contains the scalar and pseudo-scalar external sources, s and p respectively. The $f_\pm^{\mu\nu}$ are proportional to r^μ and ℓ^μ sources, which provide the vector and axial-vector external sources, $v^\mu = \frac{1}{2}(r^\mu + \ell^\mu)$ and $a^\mu = \frac{1}{2}(r^\mu - \ell^\mu)$ respectively [2, 3, 4].

The covariant derivative is given by [4, 5, 10]

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X], \quad (4)$$

with the chiral connection $\Gamma_\mu = \frac{1}{2} \{ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i \ell_\mu) u^\dagger \}$. The commutation properties of the covariant derivatives will be used in the next section [4]:

$$\begin{aligned} [\nabla^\mu, \nabla^\nu] X &= [\Gamma_{\mu\nu}, X], \\ \text{with } \Gamma_{\mu\nu} &= \frac{1}{4} [u_\mu, u_\nu] - \frac{i}{2} f_{+\mu\nu}. \end{aligned} \quad (5)$$

Every time the order of two covariant derivatives is exchanged we generate an extra operator given by $\Gamma_{\mu\nu}$, which is proportional to either v_μ and a_μ ($f_{+\mu\nu}$ term) or to at least two u^α tensors ($[u_\mu, u_\nu]$ term), i.e., $\Gamma_{\mu\nu} \sim \mathcal{O}(J) + \mathcal{O}(u^\mu u^\nu)$. All along the letter, we will denote any term proportional to at least one source $J = s, p, v^\mu, a^\mu$ as $\mathcal{O}(J)$.

Resonance chiral theory lagrangian

We will call Resonance Chiral Theory (R χ T) to the most general chiral invariant hadronic action. No *a priori* restriction is made on the number of derivatives in the lagrangian operators. Its basic building blocks X are the resonance fields $R = S, V, \dots$, the Goldstone tensors u^μ , χ_\pm , $f_\pm^{\mu\nu}$, and covariant derivatives $\nabla^{\alpha_1} \dots \nabla^{\alpha_n} X$ of any of them. A priori, symmetry does not impose any constraint on the number of derivatives or resonance fields in the operators of the lagrangian. It only determines the way how the hadronic fields must be combined [1, 5, 10].

Nevertheless, we provide in the next lines two important simplifications that rely on the freedom to redefine the hadronic fields in the generating functional $W[J]$ [10].

a.) Goldstone field redefinitions

Any R χ T lagrangian can be expressed in the general form,

$$\begin{aligned} \mathcal{L} &= \frac{F_0^2}{4} \langle u_\mu u^\mu \rangle + \langle A_S \nabla^\mu u_\mu \rangle + \langle B_S \rangle \\ &\quad - \frac{1}{2} \langle S(\nabla^2 + M_S^2) S \rangle + \Delta \mathcal{L}, \end{aligned} \quad (6)$$

with $\langle \dots \rangle$ short for trace in flavour space and where the remaining part of the lagrangian contains the terms that

will not enter in our problem:

$$\begin{aligned} \Delta \mathcal{L} &= \mathcal{O}(S^2 u^\alpha u^\beta) + \mathcal{O}(S^3) \\ &\quad + \mathcal{O}(R') + \mathcal{O}(J) + \mathcal{O}(u^\alpha u^\beta u^\mu u^\nu). \end{aligned} \quad (7)$$

The term $\mathcal{O}(S^3)$ refers to operators with at least three scalar fields and $\mathcal{O}(R')$ to terms containing at least one resonance $R' \neq S$. At least one external source is contained in the operators $\mathcal{O}(J)$. At leading order in $1/N_C$ (LO), the only operator bilinear in the scalar fields is the canonical kinetic term and those with two scalars must be either $\mathcal{O}(S^2 u^\alpha u^\beta)$ or $\mathcal{O}(J)$. Only at next-to-leading order (NLO), other S -bilinear operators will be allowed, although their presence is not relevant for the argumentation on the Goldstone field redefinition. Notice that the last two terms in Eq. (7) may contain both resonances and Goldstone fields, accounting for the remaining χ PT-like operators allowed by the symmetry.

The tensors A_S, B_S in Eq. (6) are hermitic and linear in the scalar field. For later convenience, we define them such that they cannot be included in any of the last three terms in Eq. (7), i.e., $\langle A_S \nabla^\mu u_\mu \rangle, \langle B_S \rangle \neq \mathcal{O}(R') + \mathcal{O}(J) + \mathcal{O}(u^\alpha u^\beta u^\mu u^\nu)$. We gather in the term $\langle B_S \rangle$ the linear operators in S of this kind that cannot be written like $\langle A_S \nabla^\mu u_\mu \rangle$. From this definitions, we have that both $\langle A_S \nabla^\mu u_\mu \rangle$ and $\langle B_S \rangle$ can just include one scalar field and two tensors u^μ –or any number of covariant derivatives of them–, i.e., $A_S \sim S u^\alpha$ and $B_S \sim S u^\alpha u^\beta$. Their explicit form will be provided in the next section.

We will perform at this point a Goldstone field redefinition that induces a shift in u^μ of the form

$$u_\mu \longrightarrow u_\mu + \frac{2}{F_0^2} \nabla_\mu A_S + \mathcal{O}(A_S^2). \quad (8)$$

The required Goldstone transformation is not unique. One could consider, for instance, $\xi_R \rightarrow \xi_R \exp \{-i A_S / F_0^2\}$, $\xi_L \rightarrow \xi_L \exp \{i A_S / F_0^2\}$. Notice that A_S / F_0^2 is indeed dimensionless and hermitic and the $\xi_{R,L}$ remain unitary.

This transformation produces a lagrangian with exactly the same structure in Eqs. (6)–(7) except for the term $\langle A_S \nabla^\mu u_\mu \rangle$, which is completely removed.

b.) Scalar field redefinitions

After the transformation in section a), one gets the simplified lagrangian,

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} \langle S(\nabla^2 + M_S^2) S \rangle + \langle S(\nabla^2 + M_S^2) \zeta \rangle + \langle S \eta \rangle \\ &\quad + \frac{F_0^2}{4} \langle u^\mu u_\mu \rangle + \Delta \mathcal{L}, \end{aligned} \quad (9)$$

where we have made the replacement

$$\langle B_S \rangle = \langle S(\nabla^2 + M_S^2) \zeta \rangle + \langle S \eta \rangle. \quad (10)$$

The tensor $\langle S \eta \rangle$ gathers all the terms in $\langle B_S \rangle$ which cannot be expressed like $\langle S(\nabla^2 + M_S^2) \zeta \rangle$, and the structure of $\Delta \mathcal{L}$ was defined in Eq. (7).

Now we perform the convenient field redefinition,

$$S \longrightarrow S + \zeta, \quad (11)$$

which yields the simplified lagrangian

$$\begin{aligned} \mathcal{L}' = & -\frac{1}{2} \langle S(\nabla^2 + M_S^2)S \rangle + \langle S\eta \rangle \\ & + \frac{F_0^2}{4} \langle u^\mu u_\mu \rangle + \Delta\mathcal{L}, \end{aligned} \quad (12)$$

where the term $\langle S(\nabla^2 + M_S^2)\zeta \rangle$ has been fully removed from the action. We have made use of the fact that $\eta, \zeta \sim \mathcal{O}(u^\alpha u^\beta)$.

A final detail is that beyond LO, in addition to the kinetic term, one could consider a subleading operator in the lagrangian of the form $\lambda \langle S(\nabla^2 + M_S^2)^k S \rangle$, with $k \geq 2$. The shift $S \rightarrow S + \lambda(\nabla^2 + M_S^2)^{k-1} S$ removes this term, leaving remainders of this same form at higher subleading orders. Hence, by iteration, we can always move the operators of this form to higher orders in perturbation theory, beyond any the order we are working at. Thus, the decomposition of the lagrangian given in Eqs. (6), (7) and (9) is indeed general.

General form of the $S\pi\pi$ chiral operators

In the chiral limit, all the operators that contribute to the $S \rightarrow \pi\pi$ decay amplitude are contained in the terms $\langle A_S \nabla^\mu u_\mu \rangle + \langle B_S \rangle$ of the initial lagrangian in Eq. (6). However, we have seen that the operators of the form $\langle A_S \nabla^\mu u_\mu \rangle$ and $\langle S(\nabla^2 + M_S^2)\zeta \rangle$ are not physical and they can be always removed through a convenient change in the meson field variables.

We will write now the explicit form of these operators. They cannot contain the tensors χ_\pm and $f_\pm^{\mu\nu}$ since these are proportional to external sources. Hence, they can be only composed of the tensors S and u^μ , or any number of derivatives of them. In the absence of external sources, one has that u^μ is proportional to at least one Goldstone field so our operator may contain at most two tensors of this kind. Finally, through partial integration we can always move the derivatives away from the scalar field. Hence, a general term contributing to $S \rightarrow \pi\pi$ takes the form

$$\begin{aligned} \mathcal{L}_{S \rightarrow \pi\pi} = & \lambda \langle S \{ \nabla^{\mu_1} \dots \nabla^{\mu_m} u^\rho, \nabla^{\nu_1} \dots \nabla^{\nu_n} u^\sigma \} \rangle \\ & \times t_{\mu_1, \dots, \mu_m, \rho, \nu_1, \dots, \nu_n, \sigma}, \end{aligned} \quad (13)$$

where the Lorentz tensor $t_{\mu_1, \dots, \mu_m, \rho, \nu_1, \dots, \nu_n, \sigma}$ handles all the contractions of the indices. The anticommutator $\{\dots, \dots\}$ ensures that the operator is invariant under charge and hermitian conjugations [4]. The number of covariant derivatives m, n can be any positive integer number or zero. Any reordering of the covariant derivatives $\nabla^{\mu_i} \nabla^{\mu_{i+1}}$ in $\nabla^{\mu_1} \dots \nabla^{\mu_m}$ (or similarly for $\nabla^{\nu_j} \nabla^{\nu_{j+1}}$ in $\nabla^{\nu_1} \dots \nabla^{\nu_n}$) generates an extra operator containing the chiral tensor $\Gamma^{\mu_i \mu_{i+1}}$, which does not contribute to $S \rightarrow \pi\pi$ since it contains $f_+^{\mu_i \mu_{i+1}}$ or a number of u^α tensors higher than two. Hence, we will freely change the

order of the covariant derivatives for the convenience of the derivation.

The simplest operator of this kind is

$$\mathcal{L}_{S \rightarrow \pi\pi} = \lambda \langle S \{ u^\mu, u_\mu \} \rangle = 2\lambda \langle S u^\mu u_\mu \rangle, \quad (14)$$

which is just the $c_d \langle S u^\mu u_\mu \rangle$ operator in Ref. [5].

For a higher number of derivatives, the available contractions of the Lorentz indices yield four possible cases, where one index μ_i can be contracted with ρ , with another μ_j , with some index ν_j or with σ :

1.) For $m \geq 1$, there can be at least one of the indices μ_i contracted with ρ : The covariant derivative ∇^{μ_i} can be commuted until it is placed next to u^ρ in Eq. (13). Hence, this case is equivalent to contracting the last index μ_m and ρ :

$$\begin{aligned} \mathcal{L}_{S \rightarrow \pi\pi} = & \lambda \langle S \{ \nabla^{\mu_1} \dots \nabla^{\mu_{m-1}} (\nabla^\rho u_\rho), \nabla^{\nu_1} \dots \nabla^{\nu_n} u^\sigma \} \rangle \\ & \times t_{\mu_1, \dots, \mu_{m-1}, \nu_1, \dots, \nu_n, \sigma}. \end{aligned} \quad (15)$$

Due to the $\nabla^\rho u_\rho$ tensor in the operator, this term does not contribute to the $S \rightarrow \pi\pi$ amplitude in the chiral limit when the out-going Goldstones are on-shell. Furthermore, as we saw in the previous section, this operator can be fully removed from the lagrangian through an appropriate Goldstone field redefinition.

This case is equivalent to that when $n \geq 1$ and at least one of the indices ν_j is contracted with σ .

2.) For $m \geq 2$, there can be at least one of the indices μ_i contracted with another index μ_j : The covariant derivatives ∇^{μ_i} and ∇^{μ_j} can be commuted until both of them are placed next to u^ρ . Hence, this case is equivalent to contracting the last index μ_{m-1} and μ_m :

$$\begin{aligned} \mathcal{L}_{S \rightarrow \pi\pi} = & \lambda \langle S \{ \nabla^{\mu_1} \dots \nabla^{\mu_{m-2}} \nabla^2 u^\rho, \nabla^{\nu_1} \dots \nabla^{\nu_n} u^\sigma \} \rangle \\ & \times t_{\mu_1, \dots, \mu_{m-2}, \nu_1, \dots, \nu_n, \sigma}, \end{aligned} \quad (16)$$

This can be converted into the former case by means of the identity

$$\begin{aligned} \nabla^2 u^\rho = & \nabla^\rho (\nabla^\alpha u_\alpha) - \nabla_\alpha f_-^{\alpha\rho} + [\Gamma^{\alpha\rho}, u_\alpha] \\ = & \nabla^\rho (\nabla^\alpha u_\alpha) + \mathcal{O}(J) + \mathcal{O}(u^\alpha u_\alpha u^\rho), \end{aligned} \quad (17)$$

and, therefore, it does not contribute $S \rightarrow \pi\pi$.

This case is equivalent to that when $n \geq 2$ and at least one of the indices ν_i is contracted with another ν_j .

3.) For $m, n \geq 1$, there can be at least one of the indices μ_i contracted with one of the ν_j : We can commute the covariant derivatives ∇^{μ_i} and ∇^{ν_j} and move them to the front part of the operator. Hence, this case is equivalent to contracting the first indices, μ_1 and ν_1 :

$$\begin{aligned} \mathcal{L}_{S \rightarrow \pi\pi} = & \lambda \langle S \{ \nabla^\mu \nabla^{\mu_2} \dots \nabla^{\mu_m} u^\rho, \nabla_\mu \nabla^{\nu_2} \dots \nabla^{\nu_n} u^\sigma \} \rangle \\ & \times t_{\mu_2, \dots, \mu_m, \rho, \nu_2, \dots, \nu_n, \sigma}. \end{aligned} \quad (18)$$

Through partial integration, it can be converted into the expression

$$\begin{aligned} \mathcal{L}_{S \rightarrow \pi\pi} = & \frac{\lambda}{2} [\langle \nabla^2 S \{ \nabla^{\mu_2} \dots \nabla^{\mu_m} u^\rho, \nabla^{\nu_2} \dots \nabla^{\nu_n} u^\sigma \} \rangle \\ & - \langle S \{ \nabla^2 \nabla^{\mu_2} \dots \nabla^{\mu_m} u^\rho, \nabla^{\nu_2} \dots \nabla^{\nu_n} u^\sigma \} \rangle \\ & - \langle S \{ \nabla^{\mu_2} \dots \nabla^{\mu_m} u^\rho, \nabla^2 \nabla^{\nu_2} \dots \nabla^{\nu_n} u^\sigma \} \rangle] \\ & \times t_{\mu_2, \dots, \mu_m, \rho, \nu_2, \dots, \nu_n, \sigma} \quad . \end{aligned} \quad (19)$$

The second and third terms reproduce the case no. 2 and do not contribute to $S \rightarrow \pi\pi$. The first term can be rewritten as

$$\begin{aligned} \mathcal{L}_{S \rightarrow \pi\pi} = & \frac{\lambda}{2} [-M_S^2 \langle S \{ \nabla^{\mu_2} \dots \nabla^{\mu_m} u^\rho, \nabla^{\nu_2} \dots \nabla^{\nu_n} u^\sigma \} \rangle \\ & + \langle S(\nabla^2 + M_S^2) \{ \nabla^{\mu_2} \dots \nabla^{\mu_m} u^\rho, \nabla^{\nu_2} \dots \nabla^{\nu_n} u^\sigma \} \rangle] \\ & \times t_{\mu_2, \dots, \mu_m, \rho, \nu_2, \dots, \nu_n, \sigma} \quad . \end{aligned} \quad (20)$$

The last term can be removed through the scalar field redefinition in previous section, so the only non-vanishing contribution to $S \rightarrow \pi\pi$ comes from the first term in Eq. (20). This leave us with an operator that shows again the functional structure in Eq. (13), and where the number of derivatives has been decreased in two orders. Hence, it admits to be reanalysed and eventually further simplified.

4.) In the last case remaining, for $m, n \geq 1$, there can be one of the indices μ_i contracted with σ and one of the ν_j contracted with ρ : We can commute the covariant derivatives ∇^{μ_i} and ∇^{ν_j} and move them backwards until they are placed next to u^ρ and u^σ , respectively. Hence, this case is equivalent to contracting the last indices, μ_m with σ and ν_n with ρ :

$$\begin{aligned} \mathcal{L}_{S \rightarrow \pi\pi} = & \lambda \langle S \{ \nabla^{\mu_1} \dots \nabla^{\mu_{m-1}} \nabla^\mu u^\nu, \nabla^{\nu_1} \dots \nabla^{\nu_{n-1}} \nabla_\nu u_\mu \} \rangle \\ & \times t_{\mu_1, \dots, \mu_{m-1}, \nu_1, \dots, \nu_{n-1}} \quad . \end{aligned} \quad (21)$$

One can apply the chiral tensor relation [4],

$$\nabla_\nu u_\mu = \nabla_\mu u_\nu + f_{-\mu\nu} = \nabla_\mu u_\nu + \mathcal{O}(J), \quad (22)$$

where the $\mathcal{O}(J)$ term does not contribute to $S \rightarrow \pi\pi$. Finally, moving the ∇^μ covariant derivatives to the front of the operator, we get the structure analysed in the case no. 3 in Eq. (18).

This completes the list of operators that may contribute to the $S \rightarrow \pi\pi$ decay amplitude. All non-vanishing terms can be written in the way shown in the case no. 3 and then simplified into an operator with a lower number of derivatives. By iteration, it is then possible to convert any term into the c_d operator in Eq. (14), up to contributions irrelevant for $S \rightarrow \pi\pi$.

In fact, if one admits $1/N_C$ -suppressed operators in the reasoning, there are three more available –multitrace– terms: $\langle S \rangle \langle u^\mu u_\mu \rangle$, $\langle S u^\mu \rangle \langle u_\mu \rangle$ and $\langle S \rangle \langle u^\mu \rangle \langle u_\mu \rangle$. This finally exhausts the list of $S\pi\pi$ operators, both at LO in $1/N_C$ and at subleading orders.

Conclusions

The example in this note provides a first clear example of the possibility of constructing model independent resonance chiral lagrangians. Although the action may contain an infinite number of operators, the particular vertex functions could be described through a finite number of them, making the theory predictable and model independent. In this article, the $S \rightarrow \pi\pi$ decay amplitude is determined by one single-trace operator in the chiral limit (four if we include the subleading multi-trace terms). The remaining part of the action is kept fully general both before and after simplifications.

A deeper implication of the present result refers to the structure of the loop ultraviolet divergences in the generating functional $W[J]$ [11]. In the case of the $S \rightarrow \pi\pi$ vertex function, they can only take the form of these four chiral invariant local operators. Hence, this amplitude can be rendered finite through a renormalization of c_d and the other three subleading couplings. The full theory may require an infinite number of coupling renormalizations but only a finite set of them is required for the study of particular amplitudes.

Acknowledgments

We want to acknowledge useful comments and discussions with K. Kampf, B. Moussallam, J. Novotny, J. Portolés, P. Ruiz-Femenía, M.X. Su and H.Q. Zheng. This work is supported in part by National Nature Science Foundations of China under contract number 10575002, 10421503.

* Electronic address: xiaoly@pku.edu.cn, cillero@th.phy.pku.edu.cn

- [1] C.G. Callan *et al.*, Phys. Rev. **177** (1969) 2247;
S.R. Coleman *et al.*, Phys. Rev. **177** (1969) 2239;
- [2] J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142;
Nucl. Phys. B **250** (1985) 465.
- [3] R. Kaiser & H. Leutwyler, Eur. Phys. J. C **17** (2000) 623;
P. Herrera-Siklody *et al.*, Nucl. Phys. B **497** (1997) 345.
- [4] J. Bijnens *et al.*, JHEP **9902** (1999) 020.
- [5] G. Ecker *et al.*, Nucl. Phys. B **321** (1989) 311.
- [6] G. Ecker *et al.*, Phys. Lett. B **223** (1989) 425.
- [7] A. Pich [arXiv:hep-ph/0205030].
- [8] I. Rosell *et al.*, JHEP **0408** (2004) 042.
- [9] I. Rosell *et al.*, JHEP **0701** (2007) 039;
- [10] V. Cirigliano *et al.*, Nucl. Phys. B **753** (2006) 139.
- [11] I. Rosell *et al.*, JHEP **0512** (2005) 020.
- [12] G. 't Hooft, Nucl. Phys. B **72** (1974) 461.
- [13] E. Witten, Nucl. Phys. B **160** (1979) 57.